## External Conditions which can affect delay

a) Operating Temperature
b) Supply Voltage
c) Process Variation

Drain current is proportional to $\mathrm{T}^{-1.5} \Rightarrow$ As temperature is increased, drain current is reduced for a given set of operating conditions, delay increases $\uparrow$

The temperature of the die is what counts, this is expressed as

$$
\mathrm{T}_{\mathrm{j}}=\mathrm{T}_{\mathrm{a}}+\theta_{\mathrm{ja}} \times \mathrm{P}_{\mathrm{d}}
$$

where

$$
\begin{aligned}
& \mathrm{T}_{\mathrm{a}} \equiv \text { ambient Temperature }\left({ }^{\circ} \mathrm{C}\right) \\
& \theta_{\mathrm{ja}} \equiv \text { package thermal impedance }\left({ }^{\circ} \mathrm{C} / \text { watt }\right) \\
& \mathrm{P}_{\mathrm{d}} \equiv \text { power dissipation }
\end{aligned}
$$

Typical values for $\theta_{\mathrm{ja}}$ range from 35 to $45\left({ }^{\circ} \mathrm{C} /\right.$ watt $)$, depending on chip package

| Package Type | Pin Count | $\theta_{\mathrm{ja}}$ still air | $\theta_{\mathrm{ja}} 300 \mathrm{ft} / \mathrm{min}$. | Units |
| :--- | :---: | :---: | :---: | :---: |
| Plastic J-Leaded Chip Carrier | 44 | 45 | 35 | ${ }^{\circ} \mathrm{C} / \mathrm{W}$ |
|  | 68 | 38 | 29 | ${ }^{\circ} \mathrm{C} / \mathrm{W}$ |
|  | 84 | 37 | 28 | ${ }^{\circ} \mathrm{C} / \mathrm{W}$ |
| Plastic Quad Flatpack | 100 | 48 | 40 | ${ }^{\circ} \mathrm{C} / \mathrm{W}$ |
| Very Thin (1.0mm) Quad Flatpack | 80 | 43 | 35 | ${ }^{\circ} \mathrm{C} / \mathrm{W}$ |
| Ceramic Pin Grid Array | 84 | 33 | 20 | ${ }^{\circ} \mathrm{C} / \mathrm{W}$ |
| Ceramic Quad Flatpack | 84 | 40 | 30 | ${ }^{\circ} \mathrm{C} / \mathrm{W}$ |

Parts usually characterized for different temperature ranges:

| Commercial: | $0^{\circ}$ to $70^{\circ} \mathrm{C}$ |
| :--- | :---: |
| Industrial | $-40^{\circ}$ to $85^{\circ} \mathrm{C}$ |
| Military | $-55^{\circ}$ to $125^{\circ} \mathrm{C}$ |

Voltage also affects device speed:
voltage increases $\uparrow$, drain current increases, delay decreases $\downarrow$
Typically characterize device around a power supply tolerance

## Power Supply Voltage Tolerance

| Commercial | $\pm 5 \%$ |
| ---: | :---: |
| Industrial | $\pm 10 \%$ |
| Military | $\pm 10 \%$ |

Process Variations also affect delay - wafer fabrication is a long series of chemical operations, variations in diffusion depth, dopant densities, oxide/diffusion geometry variations can cause transistor switching speeds to vary from wafer batch to wafer batch, wafer to wafer and even on the same wafer.

Transistors typically characterized as "fast", "nominal", and "slow". Need SPICE transistor models for these cases.

However, variations between $n$-speeds and $p$-speeds can be independent so one can obtain "four corners" model

| slow $n \mathrm{MOS}$ <br> fast $p \mathrm{MOS}$ | fast $n \mathrm{MOS}$ <br> fast $p \mathrm{MOS}$ |
| :--- | ---: |
|  |  |
| slow $n \mathrm{MOS}$ | fast $n \mathrm{MOS}$ <br> slow $p \mathrm{MOS}$ |

When characterizing for high speed, also want to use lowest temperature, highest voltage.

When characterizing for "slow" case, want highest temperature, lowest voltage.

| CMOS Digital Systems Checks (Commercial) |  |  |  |
| :---: | :---: | :---: | :--- |
| PROCESS | TEMPERATURE | VOLTAGE | TESTS |
| Fast- $n /$ fast- $p$ | $0^{\circ} \mathrm{C}$ | $5.5 \mathrm{~V}(3.6 \mathrm{~V})$ | Power dissipation <br> (DC), clock races |
| Slow- $n /$ slow- $p$ | $125^{\circ} \mathrm{C}$ | $4.5 \mathrm{~V}(3.0 \mathrm{~V})$ | Circuit speed, external <br> setup and hold times |
| Slow- $n /$ fast- $p$ | $0^{\circ} \mathrm{C}$ | $5.5 \mathrm{~V}(3.6 \mathrm{~V})$ | Pseudo- $n$ MOS noise <br> margin, level shifters, <br> memory write/read, |
| Fast- $n /$ slow- $p$ | $0^{\circ} \mathrm{C}$ | $5.5 \mathrm{~V}(3.6 \mathrm{~V})$ | ratioed circuits <br> Memories, ratioed <br> circuits, level shifters |

## Power Dissipation

Power Dissipation has three components:

1. Static
2. Dynamic
3. Short Circuit

For traditional CMOS design, static dissipation is limited to the leakage currents in the reversed-biased diodes formed between the substrate (or well) and source/drain regions. But in some DSM CMOS technology subthreshold leakage tends to also contribute significant static dissipation. Subthreshold leakage increases exponentially as threshold voltage decreases; i.e., lower $\mathrm{V}_{\mathrm{T}}\left(\mathrm{V}_{\mathrm{T} n}\right.$ and $\left|\mathrm{V}_{\mathrm{T} p}\right|$ ) CMOS technology has more static power dissipation (due to subthreshold leakage) than higher $\mathrm{V}_{\mathrm{T}}$ technology.

Static power dissipation can be extremely small:
1 inverter @ $5 \mathrm{~V} \Rightarrow 1$ to 2 nanowatts static power

Dynamic Power is governed by

$$
P_{d}=C_{L} V_{D D^{2} f_{p}}
$$

This is the amount of power dissipated by charging/discharging internal capacitance and load capacitance.

Note the relations:
Higher the switching speed $\Rightarrow P_{d} \uparrow$
Lower the voltage $\quad \Rightarrow P_{d} \downarrow \downarrow$ !
the Bigger the gates $\quad \Rightarrow P_{d} \uparrow$

To estimate $P_{d}$, need to know the switching frequencies of the internal signals

Typically break this into two parts:

$$
P_{d}=\left.\left(P_{d}\right)\right|_{\text {clock network }}+\left.\left(P_{d}\right)\right|_{\text {all the rest }}
$$

The power dissipation in the clock network tends to dominate in most designs. Usually assume the switching frequency of logic signals as some fraction of the clock frequency, can estimate by running some sample simulations and keeping switching statistics on internal nodes to build a probabilistic model of switching activity.

Logic synthesis techniques can be used to do the following:
a. minimize \# of gates
or b. maximize speed
and/or c. minimize switching activity

Also, have "short-circuit" power dissipation - proportional to the amount of time when both $p$ - and $n$-trees are conducting.

Slow rise/fall times on nodes can make this significant. Usually ignored in most calculations.

## Sizing Routing Calculation

The sizing of signal lines to achieve a particular RC delay was previously discussed.

For power conductors, need to worry about

1. Metal migration - too much current in too small a conductor will "blow" the conductor
2. Ground Bounce - large current spikes in $\mathrm{V}_{\mathrm{DD}} / \mathrm{GND}$ leads can occur when simultaneous outputs switch

Two components to ground bounce.
a. IR $\leftarrow \begin{aligned} & \text { for on-chip conductors, } \mathrm{R} \text { is resistance of on-chip } \\ & \text { conductor }\end{aligned}$
b. $L\left(\frac{d i}{d t}\right) \leftarrow \mathrm{L}$ is the on-chip inductance and package inductance in $\mathrm{V}_{\mathrm{DD}} / \mathrm{GND}$ pins. Package inductance dominates. Note that $\frac{d i}{d t}$ is affected by slew rates on input/output pins.

## Example

What would be the conductor width of power and ground wires to a 50 MHz clock buffer that drives 100 pF of on-chip load to satisfy the metalmigration consideration ( $\mathrm{J}_{\mathrm{AL}}=0.5 \mathrm{~mA} / \mu \mathrm{m}$ )? What is the ground bounce with chosen conductor size? The module is $500 \mu \mathrm{~m}$ from both the power and ground pads and the supply voltage is 5 volts.

1. $P=C V_{D D^{2} f}$

$$
=100 \times 10^{-12} \times 25 \times 50 \times 10^{6}
$$

$$
=125 \mathrm{~mW}
$$

$I=P / V=25 \mathrm{~mA}$
Thus the width of the clock wires should be at least $50 \mu \mathrm{~m}$. A good choice would be $100 \mu \mathrm{~m}$.
2. $R=500 / 100 \times .05$
$=5$ squares $\times .05 \Omega / \mathrm{sq}$.
$=0.25 \Omega$
$I R=0.25 \times 25 \times 10^{-3}=6.25 \mathrm{mV}$

Typically, $I R$ term of ground bounce very small compared to $L\left(\frac{d i}{d t}\right)$ term.

## Scaling

| Influence of Scaling on MOS-Device Characteristics |  |  |  |
| :--- | :---: | :---: | :---: |
| PARAMETER |  |  |  |
|  | Constant field | SCALING MODEL |  |
| Length $(L)$ | $1 / \alpha$ | $1 / \alpha$ | $1 / \alpha$ |
| Width $(W)$ | $1 / \alpha$ | $1 / \alpha$ | 1 |
| Supply voltage $(V)$ | $1 / \alpha$ | 1 | 1 |
| Gate-oxide thickness $\left(t_{o x}\right)$ | $1 / \alpha$ | $1 / \alpha$ | 1 |
| Current $\left(I=(W / L)\left(1 / t_{o x}\right) V^{2}\right)$ | $1 / \alpha$ | $\alpha$ | $\alpha$ |
| Transconductance $\left(g_{m}\right)$ | 1 | $\alpha$ | $\alpha$ |
| Junction depth $\left(X_{j}\right)$ | $1 / \alpha$ | $1 / \alpha$ | 1 |
| Substrate doping $\left(N_{A}\right)$ | $\alpha$ | $\alpha$ | 1 |
| Electric field across gate oxide $(E)$ | 1 | $\alpha$ | 1 |
| Depletion layer thickness $(d)$ | $1 / \alpha$ | $1 / \alpha$ | 1 |
| Load Capacitance $\left(C=W L / t_{o x}\right)$ | $1 / \alpha$ | $1 / \alpha$ | $1 / \alpha$ |
| Gate Delay $(V C / I)$ | $1 / \alpha$ | $1 / \alpha^{2}$ | $1 / \alpha^{2}$ |
| $)$ |  |  |  |
| DC power dissipation $\left(P_{S}\right)$ | $1 / \alpha^{2}$ | $\alpha$ | $\alpha$ |
| Dynamic power dissipation $\left(P_{d}\right)$ | $1 / \alpha^{2}$ | $\alpha$ | $\alpha$ |
| Power-delay product | $1 / \alpha^{3}$ | $1 / \alpha$ | $1 / \alpha$ |
| Gate area $(A=W L)$ | $1 / \alpha^{2}$ | $1 / \alpha^{2}$ | $1 / \alpha$ |
| Power density $(V I / A)$ | 1 | $\alpha^{3}$ | $\alpha^{2}$ |
| Current density | $\alpha$ | $\alpha^{3}$ | $\alpha^{2}$ |

Constant field scaling - all dimensions, including vertical, scaled by $\alpha$ Constant voltage scaling - constant field scaling but hold $V_{D D}$ constant Lateral - shrink gate length only

| Influence of Scaling on Interconnect | Media (Constant Field) |
| :--- | :---: |
| PARAMETERS | SCALING FACTOR |
| Line resistance $(r)$ | $\alpha$ |
| Line response $(r c)$ | 1 |
| Voltage drop | 1 |

Note: Scaling factor of 1 for the line response is bad! Actually the problem is even worse than this! You reduce the voltage to cope with power dissipation problems, you reduce current in gates and the gates do not drive the interconnect as well. Also, overall chip size is not decreasing, just putting more gates in same area so interconnect length is constant for long interconnects.

## Transient Analysis



## Static Inverter Response

A. Discharge

$$
\begin{aligned}
& \mathrm{V}_{\text {in }}\left(\mathrm{t}=0^{-}\right)=0 \rightarrow \mathrm{~V}_{\mathrm{DD}} \\
& \mathrm{~V}_{\text {out }}\left(\mathrm{t}=0^{-}\right)=\mathrm{V}_{\mathrm{DD}} \rightarrow \text { decreases }
\end{aligned}
$$


(i) Initially,

$$
\begin{array}{lr}
\mathrm{M}_{n} \text { starts out saturated } \Rightarrow & -\mathrm{C}_{\text {out }}\left(\frac{\mathrm{d} \mathrm{~V}_{\text {out }}}{\mathrm{dt}}\right)=\frac{\beta_{n}}{2}\left(\mathrm{~V}_{\mathrm{DD}}-\mathrm{V}_{\mathrm{T} n}\right)^{2} \\
\text { Integrate } \Rightarrow & \mathrm{V}_{\text {out }}(\mathrm{t})=\mathrm{V}_{\mathrm{DD}}-\frac{\beta_{n}}{2 \mathrm{C}_{\text {out }}}\left(\mathrm{V}_{\mathrm{DD}}-\mathrm{V}_{\mathrm{T} n}\right)^{2} \mathrm{t}
\end{array}
$$

This result (previous page) is valid until a time $t_{0}$ such that

$$
\begin{aligned}
\mathrm{V}_{\text {out }}\left(\mathrm{t}_{\mathrm{o}}\right) & =\mathrm{V}_{\mathrm{DD}}-\mathrm{V}_{\mathrm{T} n} \\
& =\mathrm{V}_{\mathrm{DD}}-\frac{\beta_{n}}{2 \mathrm{C}_{\mathrm{out}}}\left(\mathrm{~V}_{\mathrm{DD}}-\mathrm{V}_{\mathrm{T} n}\right)^{2} \mathrm{t}_{\mathrm{o}}
\end{aligned}
$$

So

$$
\mathrm{t}_{\mathrm{o}}=\frac{2 \mathrm{C}_{\mathrm{out}} \mathrm{~V}_{\mathrm{T} n}}{\beta_{n}\left(\mathrm{~V}_{\mathrm{DD}}-\mathrm{V}_{\mathrm{T} n}\right)^{2}}
$$

And, in terms of $t_{0}$,

$$
\begin{equation*}
\mathrm{V}_{\text {out }}(\mathrm{t})=\mathrm{V}_{\mathrm{DD}}-\mathrm{V}_{\mathrm{T} n}\left(\frac{\mathrm{t}}{\mathrm{t}_{\mathrm{o}}}\right) \tag{n}
\end{equation*}
$$

(ii) For $t \geq t_{0} \quad \Rightarrow \quad M_{n}$ is non-saturated

Here we have $\Rightarrow \quad-\mathrm{C}_{\text {out }}\left(\frac{\mathrm{dV}_{\text {out }}}{\mathrm{dt}}\right)=\frac{\beta_{n}}{2}\left[2\left(\mathrm{~V}_{\mathrm{DD}}-\mathrm{V}_{\mathrm{T} n}\right) \mathrm{V}_{\text {out }}-\mathrm{V}_{\text {out }}{ }^{2}\right]$
I.C. (initial condition) is $V_{o u t}\left(t_{0}\right)=V_{D D}-V_{T n}$, so

$$
\mathrm{V}_{\text {out }}(\mathrm{t})=\left(\mathrm{V}_{\mathrm{DD}}-\mathrm{V}_{\mathrm{T} n}\right)\left(\frac{2 \exp \left(-\mathrm{t} / \tau_{n}\right) \exp \left(\mathrm{t}_{0} / \tau_{n}\right)}{1+\exp \left(-\mathrm{t} / \tau_{n}\right) \exp \left(\mathrm{t}_{\mathrm{o}} / \tau_{n}\right)}\right)
$$

[note: $\exp \left(\mathrm{t}_{0} / \tau_{n}\right)$ is a time shift function]

$$
\mathrm{V}_{\text {out }}(\mathrm{t})=\left(\mathrm{V}_{\mathrm{DD}}-\mathrm{V}_{\mathrm{T} n}\right)\left(\frac{2 \exp \left(-\left(\mathrm{t}-\mathrm{t}_{\mathrm{o}}\right) / \tau_{n}\right)}{1+\exp \left(-\left(\mathrm{t}-\mathrm{t}_{\mathrm{o}}\right) / \tau_{n}\right)}\right)
$$

where $\quad \tau_{n}=\frac{\mathrm{C}_{\text {out }}}{\beta_{n}\left(\mathrm{~V}_{\mathrm{DD}}-\mathrm{V}_{\mathrm{T} n}\right)}=\mathrm{R}_{n} \mathrm{C}_{\text {out }}$

## Discharge "picture"



If we define $\mathrm{t}_{\mathrm{HL}}$ as $90 \%-10 \%$ time (time to discharge from $0.9 \mathrm{~V}_{\mathrm{DD}}$ to $0.1 \mathrm{~V}_{\mathrm{DD}}$ ),

$$
\mathrm{t}_{\mathrm{HL}}=\tau_{n}\left(\frac{2\left(\mathrm{~V}_{\mathrm{T} n}-\mathrm{V}_{\mathrm{o}}\right)}{\left(\mathrm{V}_{\mathrm{DD}}-\mathrm{V}_{\mathrm{T} n}\right)}+\ln \left(\frac{2\left(\mathrm{~V}_{\mathrm{DD}}-\mathrm{V}_{\mathrm{T} n}\right)}{\mathrm{V}_{\mathrm{o}}}-1\right)\right)
$$

where $\mathrm{V}_{\mathrm{o}}=0.1 \mathrm{~V}_{\mathrm{DD}}$.

Note that $\mathrm{t}_{\mathrm{HL}} \alpha \tau_{n}$.
B. Charge

$$
\begin{aligned}
& \mathrm{V}_{\text {in }}\left(\mathrm{t}=0^{-}\right)=\mathrm{V}_{\mathrm{DD}} \rightarrow 0 \\
& \mathrm{~V}_{\text {out }}\left(\mathrm{t}=0^{-}\right)=0 \rightarrow \text { increases }
\end{aligned}
$$


(i) Initially, $\mathrm{M}_{p}$ starts out saturated -

$$
\mathrm{C}_{\text {out }}\left(\frac{\mathrm{d}_{\mathrm{out}}}{\mathrm{dt}}\right)=\frac{\beta_{p}}{2}\left(\mathrm{~V}_{\mathrm{DD}}-\left|\mathrm{V}_{\mathrm{T} p}\right|\right)^{2}
$$

Integrate:

$$
\mathrm{V}_{\text {out }}(\mathrm{t})=\frac{\beta_{p}}{2 \mathrm{C}_{\text {out }}}\left(\mathrm{V}_{\mathrm{DD}}-\left|\mathrm{V}_{\mathrm{T} p}\right|\right)^{2} \mathrm{t}
$$

Valid until

$$
\begin{aligned}
\mathrm{V}_{\text {out }}\left(\mathrm{t}_{\mathrm{o}}\right)= & \left|\mathrm{V}_{\mathrm{T} p}\right|=\frac{\beta_{p}}{2 \mathrm{C}_{\mathrm{out}}}\left(\mathrm{~V}_{\mathrm{DD}}-\left|\mathrm{V}_{\mathrm{T} p}\right|\right)^{2} \mathrm{t}_{\mathrm{o}} \\
& \Rightarrow \mathrm{t}_{\mathrm{o}}=\frac{2 \mathrm{C}_{\mathrm{out}}\left|\mathrm{~V}_{\mathrm{T} p}\right|}{\beta_{p}\left(\mathrm{~V}_{\mathrm{DD}}-\left|\mathrm{V}_{\mathrm{T} p}\right|\right)^{2}}
\end{aligned}
$$

So

$$
\mathrm{V}_{\text {out }}(\mathrm{t})=\left|\mathrm{V}_{\mathrm{T} p}\right|\left(\frac{\mathrm{t}}{\mathrm{t}_{\mathrm{o}}}\right)
$$

[while $\mathrm{M}_{p}$ is sat.]
(ii) For $\mathrm{t} \geq$ to $\quad \Rightarrow \quad \mathrm{M}_{p}$ is non-saturated

$$
\mathrm{C}_{\text {out }}\left(\frac{\mathrm{d} \mathrm{~V}_{\text {out }}}{\mathrm{dt}}\right)=\frac{\beta_{p}}{2}\left(2\left(\mathrm{~V}_{\mathrm{DD}}-\left|\mathrm{V}_{\mathrm{T} p}\right|\right)\left(\mathrm{V}_{\mathrm{DD}}-\mathrm{V}_{\text {out }}\right)-\left(\mathrm{V}_{\mathrm{DD}}-\mathrm{V}_{\text {out }}\right)^{2}\right)
$$

Integrating:

$$
\begin{aligned}
& \int \frac{\mathrm{dV}_{\mathrm{out}}}{2\left(\mathrm{~V}_{\mathrm{DD}}-\left|\mathrm{V}_{\mathrm{T} p}\right|\right)\left(\mathrm{V}_{\mathrm{DD}}-\mathrm{V}_{\mathrm{out}}\right)-\left(\mathrm{V}_{\mathrm{DD}}-\mathrm{V}_{\mathrm{out}}\right)^{2}}=\frac{\beta_{p}}{2 \mathrm{C}_{\mathrm{out}}} \int \mathrm{dt} \\
& \text { Helpful to define } \quad \begin{aligned}
\quad & \equiv \mathrm{V}_{\mathrm{DD}}-\mathrm{V}_{\mathrm{out}} \\
\mathrm{~d} v & \equiv-\mathrm{dV}_{\mathrm{out}}
\end{aligned}
\end{aligned}
$$

Then

$$
-\int \frac{\mathrm{dv}}{2\left(\mathrm{~V}_{\mathrm{DD}}-\left|\mathrm{V}_{\mathrm{T} p}\right|\right) v-v^{2}}=\frac{\beta_{p}}{2 \mathrm{C}_{\mathrm{out}}} \int \mathrm{dt}
$$

This form is now similar to the discharge case.
Integrate to get

$$
\mathrm{V}_{\text {out }}(\mathrm{t})=\left(\mathrm{V}_{\mathrm{DD}}-\left(\mathrm{V}_{\mathrm{DD}}-\left|\mathrm{V}_{\mathrm{T} p}\right|\right) \frac{2 \exp \left(-\left(\mathrm{t}-\mathrm{t}_{\mathrm{o}}\right) / \tau_{p}\right)}{1+\exp \left(-\left(\mathrm{t}-\mathrm{t}_{\mathrm{o}}\right) / \tau_{p}\right)}\right)
$$

where

$$
\tau_{p}=\frac{\mathrm{C}_{\mathrm{out}}}{\beta_{p}\left(\mathrm{~V}_{\mathrm{DD}}-\left|\mathrm{V}_{\mathrm{T} p}\right|\right)}=\mathrm{R}_{p} \mathrm{C}_{\mathrm{out}}
$$

## Charging "picture"



If we define $\mathrm{t}_{\mathrm{LH}}$ as $10 \%-90 \%$ time (time to charge from $0.1 \mathrm{~V}_{\mathrm{DD}}$ to $0.9 \mathrm{~V}_{\mathrm{DD}}$ ),

$$
\mathrm{t}_{\mathrm{LH}}=\tau_{p}\left(\frac{2\left(\left|\mathrm{~V}_{\mathrm{T} p}\right|-\mathrm{V}_{\mathrm{o}}\right)}{\left(\mathrm{V}_{\mathrm{DD}}-\left|\mathrm{V}_{\mathrm{T} p}\right|\right)}+\ln \left(\frac{2\left(\mathrm{~V}_{\mathrm{DD}}-\left|\mathrm{V}_{\mathrm{T} p}\right|\right)}{\mathrm{V}_{\mathrm{o}}}-1\right)\right)
$$

where $\mathrm{V}_{\mathrm{o}}=0.1 \mathrm{~V}_{\mathrm{DD}}$.

Note that tLH $\alpha \tau_{p}$.

## Maximum Switching Frequency

A gate's minimum time requirement to undergo a complete switching cycle is $\left(\mathrm{t}_{\mathrm{HL}}+\mathrm{t}_{\mathrm{LH}}\right)$. The maximum switching frequency for a gate is

$$
\mathrm{f}_{\max }=\frac{1}{\mathrm{t}_{\mathrm{HL}}+\mathrm{t}_{\mathrm{LH}}} .
$$

## Propagation Delay

Propagation delay time, tp, conveniently describes the logic delay through a gate.

$$
\mathrm{t}_{\mathrm{P}}=\frac{1}{2}\left(\mathrm{t}_{\mathrm{HL}}+\mathrm{t}_{\mathrm{LH}}\right)
$$

where

$$
\mathrm{tpHL}=\tau_{n}\left(\frac{2 \mathrm{~V}_{\mathrm{T} n}}{\left(\mathrm{~V}_{\mathrm{DD}}-\mathrm{V}_{\mathrm{T} n}\right)}+\ln \left(\frac{4\left(\mathrm{~V}_{\mathrm{DD}}-\mathrm{V}_{\mathrm{T} n}\right)}{\mathrm{V}_{\mathrm{DD}}}-1\right)\right)
$$

and

$$
\operatorname{tPLH}=\tau_{p}\left(\frac{2\left|\mathrm{~V}_{\mathrm{T} p}\right|}{\left(\mathrm{V}_{\mathrm{DD}}-\left|\mathrm{V}_{\mathrm{T} p}\right|\right)}+\ln \left(\frac{4\left(\mathrm{~V}_{\mathrm{DD}}-\left|\mathrm{V}_{\mathrm{T} p}\right|\right)}{\mathrm{V}_{\mathrm{DD}}}-1\right)\right) .
$$

Here, tpHL and tPLH, are the propagation delays for a high-to-low and a low-tohigh transition, respectively. tpHL is the time required for the output to change from $\mathrm{V}_{\mathrm{DD}}$ to $\mathrm{V}_{t h}$ (for the above equations, $\mathrm{V}_{t h}=\left(\mathrm{V}_{\mathrm{DD}} / 2\right)$ is assumed). Likewise, tPLH is the time needed for a gate's output to rise from $\mathrm{V}_{\mathrm{OL}}$ to $\mathrm{V}_{t h}$.

Physical interpretation of $\operatorname{tp} \Rightarrow$ average time for a gate's output to respond to a logic state change at its input .

